Prediction of Burning Rates in Nozzleless Rocket Motors

Merrill K. King*
Atlantic Research Corporation, Alexandria, Virginia

Abstract

AN algorithm for prediction of composite propellant burning rate as a function of pressure, crossflow velocity, port diameter, and time derivatives of pressure and crossflow velocity, under conditions of high product crossflow, has been developed. In this algorithm, zero-crossflow burn rate vs pressure data for the formulation of interest are used as input. Correlations of steady-state erosive burning rate ratio (as calculated from an existing complex erosive burning computer code) as a function of the first three parameters, zero-crossflow burn rate, and effective flame temperature are then used to calculate steady-state burn rate. Finally, corrections for transient burning phenomena accompanying rapid changes in operating pressure or crossflow velocity are made using a modified P-dot formula.

Contents

In the interior ballistics analysis section of the Nozzleless Performance Program (NPP), developed for prediction of nozzleless motor behavior, source terms related to local propellant burning rate are required for the mass, momentum, and energy equations at each axial location along the grain port. The high product crossflow velocities associated with nozzleless motor operation can result in marked augmentation of the propellant burning rate (erosive burning). These effects are generally not scale-independent, decreasing with increasing grain port diameter. Finally, time-gradients of pressure and crossflow velocity are often sufficiently high that transient effects must be treated. The objective of this work was development of a compact algorithm for determination of propellant burning rate at each location (node) and time during operation of a nozzleless motor, given pressure, crossflow velocity, port diameter, and rate of change of pressure and velocity.

The general approach taken is first to calculate the quasi-steady-state burn rate ($\bar{r}$) and then calculate a correction term for unsteady-state effects to yield instantaneous burn rate ($r=\bar{r}+r'$). The quasi-steady rate is calculated as the product of zero-crossflow rate ($\bar{r}_0$), empirically input as a function of pressure by the user, and an erosive burning ratio ($\bar{r}/\bar{r}_0$). An erosive burning model for ammonium-perchlorate-oxidized composite propellants (metalized and non-metalized) which fits experimental results over a large range of parameters, along with correctly predicting scaling trends, has been developed.[1-5] It is felt that this model can be used with reasonable confidence for the calculation in NPP of erosive burning ratio as a function of pressure, velocity, port diameter, and propellant parameters. However, direct coupling of the complete erosive burning model into NPP would lead to prohibitive computer run times and costs. Therefore, parametric runs with this code were used to develop correlating equations for incorporation into NPP.

First, effects of scaling (port diameter) were examined. For any given propellant, pressure, and velocity, it was found that the predicted $\bar{r}/\bar{r}_0$ could be related to channel diameter by

$$\bar{r}/\bar{r}_0\,|_D = \bar{r}/\bar{r}_0\,|_{D=D_{\text{reference}}} - B\ln(D/D_{\text{reference}}) \tag{1}$$

Moreover, B correlated as a function of the reference $\bar{r}/\bar{r}_0$ and an effective flame temperature. (For nonmetalized propellants, effective flame temperature is the actual flame temperature, while for metalized formulations $T_{\text{eff}} = T^* + 10(\text{wt \% metal})$, T^*(K) being the flame temperature with no metal combustion). This procedure led to (see fig. 1):

$$B = G\sqrt{T_{\text{eff}}}/100 \tag{2}$$

where

$$G = -0.85 + 0.85(\bar{r}/\bar{r}_0\,|_{D=0.1}) \quad \text{for } \bar{r}/\bar{r}_0\,|_{D=0.1} \le 1.2 \tag{3}$$

$$G = -0.0332 + 0.1694(\bar{r}/\bar{r}_0\,|_{D=0.1}) \quad \text{for } \bar{r}/\bar{r}_0\,|_{D=0.1} > 1.2 \tag{4}$$

$$\bar{r}/\bar{r}_0\,|_D = \bar{r}/\bar{r}_0\,|_{D=0.1} - B\ln(10D) \tag{5}$$

Next, attention was turned to developing a correlation for $\bar{r}/\bar{r}_0\,|_{D=0.1\,\text{ft}}$ as a function of pressure, velocity, and propellant parameters. A large number of calculations were performed with the full model, covering a wide range of pressures, crossflow velocities, and propellant types. Fortuitously, it was found that for any given pressure and crossflow velocity, erosive burning ratio could be correlated almost perfectly with just two propellant parameters, base (no-crossflow) burning rate, and effective flame temperature, with the latter parameter being much less influential than the former. Careful study revealed that $\bar{r}/\bar{r}_0$ could be fit quite well (Fig. 2) by:

$$\bar{r}/\bar{r}_0\,|_{D=0.1} = A_1/\bar{r}_0^{\,A_2} \tag{6}$$

Table 1 Tabulation of $k_1 - k_6$ [Eqs. (7-8)] vs temperature

T_{eff}(K)	k_1	k_2
1667	86.3	0.929
2017	10.93	0.577
2534	2.02	0.279
2974	0.805	0.139

T_{eff}(K)	V-range	k_3	k_4	k_5	k_6
1667	>2000	0.00255	0.457	0.913	−0.0217
	<2000	0.258	−0.151	−0.0894	+0.1103
2017	>1500	0.100	0.040	0.0124	0.0873
	<1500	0.363	−0.136	−0.267	0.1255
2534	All	0.0973	0.100	−0.091	0.089
	—	—	—	—	—
2974	>700	0.0131	0.378	0.345	0.0287
	<700	1.122	−0.30	−0.622	0.176

Received June 8, 1982; presented as Paper 82-1200 at the AIAA/SAE/ASME 18th Joint Propulsion Conference, Cleveland, Ohio, June 21-23, 1982; synoptic received April 12, 1984. Full paper available from the AIAA Library, 555 West 57th St., New York, N.Y. 10019; microfiche—$4.00, hard copy—$9.00. **Remittance must accompany order.**

*Chief Scientist, Research and Technology Division. Member AIAA.

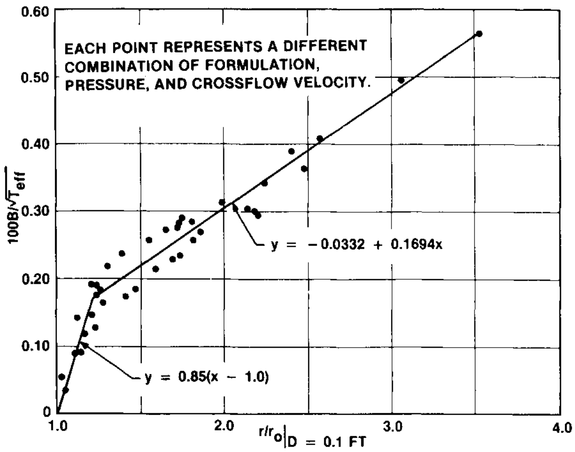

Fig. 1 Dependence of $G = 100\,B/\sqrt{T_{eff}}$ on reference erosive burning rate ratio.

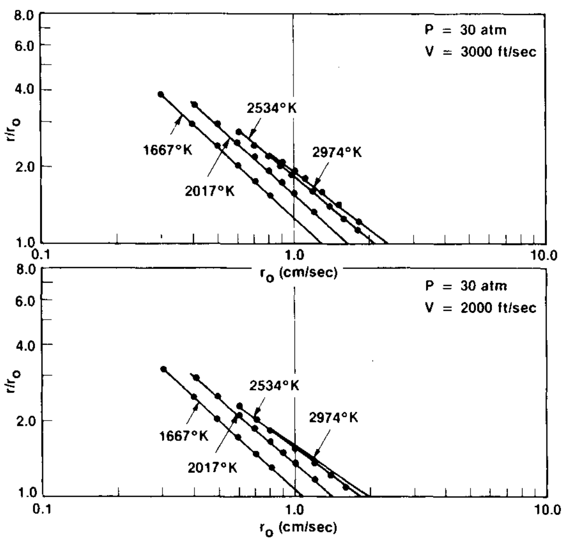

Fig. 2 Typical dependence on reference erosive burning rate ratio on base burn rate and effective temperature at fixed pressure and crossflow velocity.

where A_1 is a function of crossflow velocity and effective flame temperature and A_2 is a function of these two parameters and velocity. Closed-form correlations of A_1 and A_2 as functions of P and V were developed for several flame temperatures.

$$A_1 = 1 - k_1 V^{-k_2} \tag{7}$$

$$A_2 = k_3 V^{k_4} p [k_5 + k_6 \ln(V)] \tag{8}$$

where the constants k_1-----k_6 are functions of flame temperature (Table 1).

The following procedure thus may be used to calculate erosive burning ratio and total steady-state burn rate for specified pressure, velocity, channel (port) diameter and propellant. First, logarithmic interpolation of a base (no-crossflow) burning rate vs pressure table is used to obtain base burning rate, $\bar{r}_0$. Next, A_1 and A_2 are evaluated for tabular values of flame temperature bracketing the actual value using Eqs. (7) and (8), and $\bar{r}/\bar{r}_0|_{D=0.1}$ values are calculated for these bracketing values using Eq. (6). Linear interpolation next is used to obtain the $\bar{r}/\bar{r}_0|_{D=0.1}$ value at the actual temperature, and Eqs. (2)-(5) are then used to correct for $D \neq 0.1$ ft. (Due to extrapolation difficulties, an alternate procedure is used for calculation of $\bar{r}/\bar{r}_0$ at $V < 400$ ft/s.[6]) If the calculated value of $\bar{r}/\bar{r}_0$ is less than 1.0, it is defaulted to unity. Finally, erosive ratio and base burn rate are multiplied to give total steady-state burning rate.

Several approaches were considered for treatment of alteration of burning rates from steady-state values due to thermal profile relaxation effects during periods of rapidly changing pressure of crossflow velocity.[6] The P-dot approach of von Elbe[7] was selected as a reasonable compromise between completeness and compactness, yielding:

$$r = \bar{r}(P,V,D) + \frac{N\alpha}{\bar{r}^2}\left[\frac{\partial \bar{r}}{\partial P}\bigg|_{V}\frac{dP}{dt} + \frac{\partial \bar{r}}{\partial V}\bigg|_{P}\frac{dV}{dt}\right] \tag{9}$$

Thus, for correction of the steady-state burn rate for transient effects, the following straightforward approach is employed. Subsequent to calculation of $\bar{r}$ at the condition of interest with the steady-state algorithm, this same algorithm is used to calculate $\bar{r}$ at the same pressure and slightly higher and lower velocities, and at the same velocity and slightly higher and lower pressures, and thus to calculate the partial derivatives. Equation (9) is then used with values of dV/dt and dP/dt from the interior ballistics module and user input values of propellant thermal diffusivity and N (default value of 2.0) to calculate instantaneous burning rate r.

References

[1]King, M. K., "Predicted and Measured Effects of Pressure and Crossflow Velocity on Composite Propellant Burning rate," CPIA Publication 329, Vol. I., 17th JANNAF Combustion Meeting, Nov. 1980, p. 99.

[2]King, M. K., "Experimental and Theoretical Study of the Effects of Pressure and Crossflow Velocity on Composite Propellant Burning Rate," *18th International Symposium on Combustion,* The Combustion Institute, Pittsburgh, Pa., 1981, pp. 207-216.

[3]King, M. K., "A Model for the Effects of Pressure and Crossflow Velocity on Composite Propellant Burning Rate," AIAA Paper 79-1171, June 1979.

[4]King, M. K., "Model for Steady-State Combustion of Unimodal Composite Solid Propellants," AIAA Paper 78-216, Jan. 1978.

[5]King, M. K., "Erosive Burning of Composite Solid Propellants," CPIA Publication 297, Vol. II, *15th JANNAF Combustion Meeting,* Feb. 1979, pp. 179-188.

[6]King, M. K., "Prediction of Solid Propellant Burning Rates in Nozzleless Motor," AIAA Paper 82-1200, June 1982.

[7]Von Elbe, G., "Theory of Solid Propellant Ignition and Response to Pressure Transients," Nineteenth Interagency Solid Propulsion Meeting, CPIA Publication No. 18, Vol. III, 1963, p. 95.